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$$R_A = P \uparrow$$

$$M_A = Pa \circlearrowleft$$

From overall equilibrium:
$$R_A = P \uparrow$$
 $M_A = Pa \circlearrowleft$

(a) $EIv'' = M_r = Px - Pa$ Boundary Conditions:
$$EIv' = \frac{Px^2}{2} - Pax + C_1$$
 At $x = 0$, $v' = 0$: $C_1 = 0$

At
$$x = 0$$
, $v' = 0$: $C_1 = 0$

$$EIv = \frac{Px^3}{6} - \frac{Pax^2}{2} + C_1x + C_2$$

At
$$x = 0$$
, $v = 0$: $C_2 = 0$

(b)
$$\delta_B = v_{x=L} = \frac{P}{6EI} (L^3 - 3aL^2)$$

For $a = a_{\min} = \frac{L}{4}$: $\delta_B = \frac{+PL^3}{24EI} = \frac{PL^3}{24EI} \uparrow$
For $a = a_{\max} = \frac{3L}{4}$: $\delta_B = \frac{-5PL^3}{24EI} = \frac{5PL^3}{24EI} \downarrow$ $a = \frac{3L}{4}$:

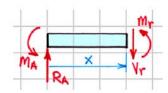
For
$$a = a_{\min} = \frac{L}{4}$$
:

$$\delta_B = \frac{+PL^3}{24EI} = \frac{PL^3}{24EI} \uparrow$$

For
$$a = a_{\text{max}} = \frac{3L}{4}$$
:

$$\delta_B = \frac{-5PL^3}{24EI} = \frac{5PL^3}{24EI} \downarrow$$

$$v = 0 \text{ when } L^3 - 3aL^2 = 0$$

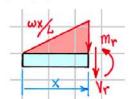


$$a = L/3$$
 Ans.

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From overall equilibrium:

$$R_B = R_C = (3wL/4) \uparrow$$





$$EIv_1'' = M_r = \frac{-wx^3}{18L}$$

$$EIv_1' = \frac{-wx^4}{72I} + C_1$$

$$EIv_1 = \frac{-wx^5}{360L} + C_1x + C_2$$

$$L \le x \le 3L$$
:
 $EIv_2'' = M_r = \frac{-wx^3}{18L} + \frac{3wL(x-L)}{4}$

$$EIv_2' = \frac{-wx^4}{72L} + \frac{3wL(x-L)^2}{8} + C_3$$

$$EIv_1 = \frac{-wx^5}{360L} + C_1x + C_2$$

$$EIv_2 = \frac{-wx^5}{360L} + \frac{3wL(x-L)^3}{24} + C_3x + C_4$$

Boundary Conditions:

At
$$x = L$$
, $v_1 = 0$: $C_1 L + C_2 = \frac{wL^5}{360L}$ (a)

At
$$x = L$$
, $v_2 = 0$: $C_3 L + C_4 = \frac{wL^5}{360L}$ (b)

At
$$x = 3L$$
, $v_2 = 0$: $C_3 3L + C_4 = \frac{-117wL^4}{360}$ (c)

Matching Condition:

At
$$x = L$$
, $v'_1 = v'_2$:
$$\frac{-wL^3}{72} + C_1 = \frac{-wL^3}{72} + C_3$$
 (d)

Solving Eqs. (a), (b), (c), and (d) gives
$$C_1 = C_3 = \frac{-59wL^3}{360}$$
 $C_2 = C_4 = \frac{wL^4}{6}$

(a)
$$0 \le x \le L$$
: $v_1 = \frac{w}{360EIL} \left(-x^5 - 59L^4x + 60L^5 \right)$

$$\delta_A = v_{1,x=0} = \frac{+60wL^5}{360EIL} = \frac{wL^4}{6EI} \uparrow$$
 Ans.

(b)
$$L \le x \le 3L$$
: $v_2 = \frac{w}{360EIL} \left[-x^5 + 45L^2 (x - L)^3 - 59L^4x + 60L^5 \right]$

$$\delta_M = v_{2,x=2L} = \frac{w}{360EIL} \left[-32L^5 + 45L^5 - 118L^5 + 60L^5 \right]$$

$$\delta_M = \frac{-wL^4}{8EIL} = \frac{wL^4}{8EI} \downarrow \dots$$
 Ans.

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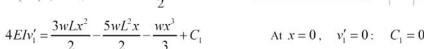
From overall equilibrium:

$$R_A = 3wL \uparrow$$

$$M_A = (5wL^2/2)$$
 \circlearrowleft

 $0 \le x \le L$:

$$E(4I)v_1'' = M_r = 3wLx - \frac{5wL^2}{2} - wx^2$$
 Boundary Conditions:



At
$$x = 0$$
, $v_1' = 0$: $C_1 = 0$

$$4EIv_1 = \frac{wLx^3}{2} - \frac{5wL^2x^2}{4} - \frac{wx^4}{12} + C_1x + C_2 \qquad \text{At } x = 0, \quad v_1 = 0: \quad C_2 = 0$$

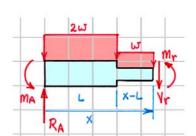
At
$$x = 0$$
, $v_1 = 0$: $C_2 = 0$

 $L \le x \le 2L$:

$$EIv_2'' = M_r = 3wLx - \frac{5wL^2}{2} - wx^2 + \frac{w(x-L)^2}{2}$$

$$EIv_2' = \frac{3wLx^2}{2} - \frac{5wL^2x}{2} - \frac{wx^3}{3} + \frac{w(x-L)^3}{6} + C_3$$

$$EIv_2 = \frac{wLx^3}{2} - \frac{5wL^2x^2}{4} - \frac{wx^4}{12} + \frac{w(x-L)^4}{24} + C_3x + C_4$$



At
$$x = L$$
, $v'_1 = v'_2$: $\frac{1}{4} \left[\frac{3wL^3}{2} - \frac{5wL^3}{2} - \frac{wL^3}{3} \right] = \frac{3wL^3}{2} - \frac{5wL^3}{2} - \frac{wL^3}{3} + C_3$

At
$$x = L$$
, $v_1 = v_2$: $\frac{1}{4} \left[\frac{wL^4}{2} - \frac{5wL^4}{4} - \frac{wL^4}{12} \right] = \frac{wL^4}{2} - \frac{5wL^4}{4} - \frac{wL^4}{12} + C_3L + C_4$

$$C_3 = wL^3$$

$$C_4 = -3wL^4/8$$

(a)
$$v_1 = \frac{w}{48EI} \left(-x^4 + 6Lx^3 - 15L^2x^2 \right)$$

$$\delta_B = v_{1,x=L} = \frac{w}{48EI} \left(-L^4 + 6L^4 - 15L^4 \right) = \frac{-10wL^4}{48EI} = \frac{5wL^4}{24EI} \downarrow \dots Ans.$$

(b)
$$v_2 = \frac{w}{24EI} \left[-2x^4 + 12Lx^3 - 30L^2x^2 + (x-L)^4 + 24L^3x - 9L^4 \right]$$

$$\delta_C = v_{2,x=2L} = \frac{w}{24EL} \left[-2(16L^4) + 12(8L^4) - 30(4L^4) + (L^4) + 24(2L^4) - 9L^4 \right]$$

$$\delta_C = \frac{-16wL^4}{24EI} = \frac{2wL^4}{3EI} \downarrow \dots$$
 Ans.

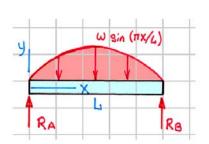
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$$EIv'''' = -w\sin\frac{\pi x}{L}$$

$$EIv'''' = \frac{wL}{\pi}\cos\frac{\pi x}{L} + C_1$$

$$EIv'' = \frac{wL^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$$EIv' = \frac{-wL^3}{\pi^3}\cos\frac{\pi x}{L} + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$EIv = \frac{-wL^4}{\pi^4} \sin \frac{\pi x}{L} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$



Boundary Conditions:

At
$$x = 0$$
, $M = EIv'' = 0$:

$$C_{2} = 0$$

At
$$x = L$$
, $M = EIv'' = 0$: $C_1 = 0$

$$C_{1} = 0$$

At
$$x = 0$$
, $v = 0$:

$$C_4 = 0$$

At
$$x = L$$
, $v = 0$:

$$C_3 = 0$$

(a)
$$v = \frac{-wL^4}{\pi^4 EI} \sin \frac{\pi x}{L}$$
 Ans.

(b)
$$\delta_M = v_{x=L/2} = \frac{-wL^4}{\pi^4 EI} = \frac{wL^4}{\pi^4 EI} \downarrow$$
 Ans.

(c)
$$\theta_A = v'_{x=0} = \frac{-wL^3}{\pi^3 EI} = \frac{wL^3}{\pi^3 EI} \mathbf{S}$$
 Ans.

(d)
$$R_A = V_{x=0} = EIv_{x=0}^m = \frac{wL}{\pi} = \frac{wL}{\pi} \uparrow \dots$$
 Ans.

$$R_{\scriptscriptstyle B} = -V_{\scriptscriptstyle x=L} = -EIv_{\scriptscriptstyle x=L}^{\prime\prime\prime} = -\left[\frac{-wL}{\pi}\right] = \frac{+wL}{\pi} = \frac{wL}{\pi} \uparrow \qquad \text{Ans.}$$