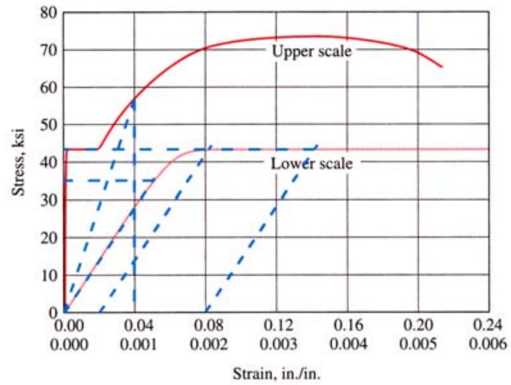
4-5

$$A_o = \pi(0.25)^2/4 = 0.04909 \text{ in.}^2$$

$$A_f = \pi(0.212)^2/4 = 0.03530 \text{ in.}^2$$

From the $\sigma - \varepsilon$ diagram:



(a) $E = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{34.5-0}{0.00125-0} = 27{,}600 \text{ ksi}$ ....**Ans.**

(b) $\sigma_{PL} \cong 36 \text{ ksi}$ ....**Ans.**

(c) $\sigma_{ult} \cong 73 \text{ ksi}$ ....**Ans.**

(d) $\sigma_{ys}(0.05\%) \cong 43 \text{ ksi}$ .... **Ans.**

(e) $\sigma_{ys}(0.20\%) \cong 43 \text{ ksi}$ .... **Ans.**

(f) $\sigma_t \cong 65 \text{ ksi}$ .... **Ans.**

(g) $\sigma_{ft} = \dfrac{P_f}{A_f} = \dfrac{\sigma_f A_o}{A_f} \cong \dfrac{65(0.04909)}{0.03530} = 90 \text{ ksi}$ .... **Ans.**

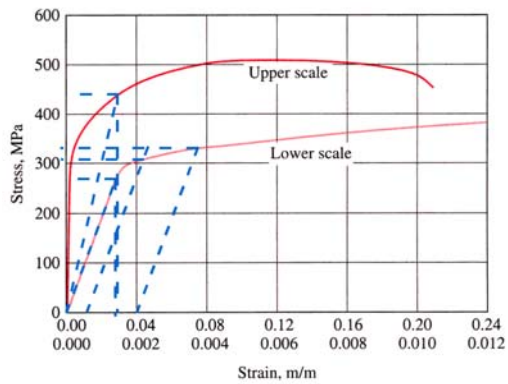
(h) $E_t = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{64-50}{0.06-0.03} = 467 \text{ ksi}$ .... **Ans.**

(i) $E_s = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{56-0}{0.04} = 1400 \text{ ksi}$ .... **Ans.**

4-6

$$A_o = \pi(5.64)^2/4 = 24.98 \text{ mm}^2 = 24.98(10^{-6}) \text{ m}^2$$

$$A_f = \pi(4.75)^2/4 = 17.72 \text{ mm}^2 = 17.72(10^{-6}) \text{ m}^2$$



From the $\sigma - \varepsilon$ diagram:

(a) $$E = \frac{\Delta\sigma}{\Delta\varepsilon} \cong \frac{(225-0)(10^6)}{0.0012-0}$$

$E \cong 187(10^9) \text{ N/m}^2 = 187 \text{ GPa}$ .... **Ans.**

(b) $\sigma_{PL} \cong 270 \text{ MPa}$ .... **Ans.**

(c) $\sigma_{ult} \cong 510 \text{ MPa}$ .... **Ans.**

(d) $\sigma_{ys}(0.05\%) \cong 305 \text{ MPa}$ .... **Ans.**

(e) $\sigma_{ys}(0.20\%) \cong 328 \text{ MPa}$ .... **Ans.**

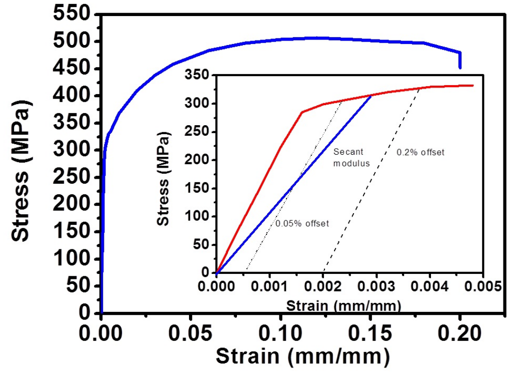
(f) $\sigma_t \cong 450 \text{ MPa}$ .... **Ans.**

(g) $\sigma_{ft} = \dfrac{P_f}{A_f} = \dfrac{\sigma_f A_o}{A_f} \cong \dfrac{450(24.98)}{17.72} = 634 \text{ MPa}$ .... **Ans.**

(h) $E_t = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{(460-410)(10^6)}{0.04-0.02} = 2.50(10^9) \text{ N/m}^2 = 2.50 \text{ GPa}$ .... **Ans.**

(i) $E_s = \dfrac{\Delta\sigma}{\Delta\varepsilon} \cong \dfrac{(440-0)(10^6)}{0.03} = 14.67(10^9) \text{ N/m}^2 = 14.67 \text{ GPa}$ .... **Ans.**

4-8



$$A_o = \pi(11.28)^2/4 = 99.93 \text{ mm}^2$$

$$A_f = \pi(9.50)^2/4 = 70.88 \text{ mm}^2$$

First calculate stresses and strains from the given data and draw the $\sigma - \varepsilon$ diagram (next page). Then, from the $\sigma - \varepsilon$ diagram:

(a) $$E = \frac{\Delta\sigma}{\Delta\varepsilon} \cong \frac{(222-0)(10^6)}{0.0012-0}$$

$E \cong 185(10^9) \text{ N/m}^2 = 185 \text{ GPa}$ .... **Ans.**

(b) $\sigma_{PL} \cong 270 \text{ MPa}$ .... **Ans.**

(c) $\sigma_{ult} \cong 510 \text{ MPa}$ .... **Ans.**

(d) $\sigma_{ys}(0.05\%) \cong 305 \text{ MPa}$ .... **Ans.**

(e) $\sigma_{ys}(0.20\%) \cong 328 \text{ MPa}$ .... **Ans.**

(f) $\sigma_t \cong 450 \text{ MPa}$ .... **Ans.**

(g) $\sigma_{ft} = \frac{P_f}{A_f} = \frac{\sigma_f A_o}{A_f} \cong \frac{450(99.93)}{70.88} = 634 \text{ MPa}$ .... **Ans.**

(h) $E_t = \frac{\Delta\sigma}{\Delta\varepsilon} \cong \frac{80(10^6)}{0.0048} = 16.7(10^9) \text{ N/m}^2 = 16.7 \text{ GPa}$ .... **Ans.**

(i) $E_s = \frac{\Delta\sigma}{\Delta\varepsilon} \cong \frac{315(10^6)}{0.0029} = 109(10^9) \text{ N/m}^2 = 109 \text{ GPa}$ .... **Ans.**

4-16*

The given values are $E = 73$ GPa $\nu = 0.33$

$$\varepsilon_a = \varepsilon_x = 875\ \mu\text{m/m} \qquad \varepsilon_b = \varepsilon_{120°} = 700\ \mu\text{m/m} \qquad \varepsilon_c = \varepsilon_{60°} = -650\ \mu\text{m/m}$$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_b = (875)\cos^2(120°) + \varepsilon_y \sin^2(120°) + \gamma_{xy}\sin(120°)\cos(120°) = 700$$

$$\varepsilon_c = (875)\cos^2(60°) + \varepsilon_y \sin^2(60°) + \gamma_{xy}\sin(60°)\cos(60°) = -650$$

$$0.75000\varepsilon_y - 0.43301\gamma_{xy} = 481.25$$

$$0.75000\varepsilon_y + 0.43301\gamma_{xy} = -868.75$$

$$\varepsilon_y = -258.33\ \mu\text{m/m} \qquad \gamma_{xy} = -1558.85\ \mu\text{rad}$$

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) = \frac{\left(73\times10^3\right)}{1-\left(0.33\right)^2}\left[(875) + 0.33(-258.33)\right]\left(10^{-6}\right)$$

$\sigma_x = +64.7$ MPa $= 64.7$ MPa (T) .... **Ans.**

$$\sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) = \frac{\left(73\times10^3\right)}{1-\left(0.33\right)^2}\left[(-258.33) + 0.33(875)\right]\left(10^{-6}\right)$$

$\sigma_y = +2.49$ MPa $= 2.49$ MPa (T) .... **Ans.**

$$G = \frac{E}{2(1+\nu)} = \frac{73}{2(1+0.33)} = 27.444\ \text{GPa}$$

$\tau_{xy} = G\gamma_{xy} = \left(27.444\times10^3\right)\left(-1558.85\times10^{-6}\right) = -42.8$ MPa .... **Ans.**

4-23

The given values are $E = 10,600$ ksi $\nu = 0.33$

$\varepsilon_a = \varepsilon_x = 875\ \mu$in./in. $\varepsilon_b = \varepsilon_{135°} = 700\ \mu$in./in. $\varepsilon_c = \varepsilon_{-135°} = -350\ \mu$in./in.

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_b = (875)\cos^2(135°) + \varepsilon_y \sin^2(135°) + \gamma_{xy}\sin(135°)\cos(135°) = 700$$

$$\varepsilon_c = (875)\cos^2(-135°) + \varepsilon_y \sin^2(-135°) + \gamma_{xy}\sin(-135°)\cos(-135°) = -350$$

$$0.5000\varepsilon_y - 0.5000\gamma_{xy} = 262.5$$

$$0.5000\varepsilon_y + 0.5000\gamma_{xy} = -787.5$$

$$\varepsilon_y = -525.00\ \mu\text{in./in.}$$

$$\gamma_{xy} = -1050.00\ \mu\text{rad}$$

(a) $$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(-1050)}{(875)-(-525)} = -18.435°,\quad 71.565°$$

When $\theta_p = -18.435°$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= (875)\cos^2\theta_p + (-525)\sin^2\theta_p + (-1050)\sin\theta_p\cos\theta_p$$

$$= 1050.00\ \mu\text{in./in.} = \varepsilon_{p1}$$

$$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = -700.00\ \mu\text{in./in.}$$

$\varepsilon_{p1} = 1050\ \mu$in./in. ⦩ 18.43° .... **Ans.**

$\varepsilon_{p2} = -700\ \mu$in./in. ⦨ 71.57° .... **Ans.**

$\varepsilon_{p3} = \dfrac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = \dfrac{-0.33}{1-0.33}[(875)+(-525)] = -172.4\ \mu$in./in. .... **Ans.**

$\gamma_{max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1750\ \mu$rad .... **Ans.**

(b) $$\sigma_{p1} = \frac{E}{1-\nu^2}(\varepsilon_{p1} + \nu\varepsilon_{p2}) = \frac{(10,600)}{1-(0.33)^2}[(1050)+0.33(-700)](10^{-6})$$

$\sigma_{p1} = +9.7423$ ksi $\cong$ 9.74 ksi (T) .... **Ans.**

$$\sigma_{p2} = \frac{E}{1-\nu^2}(\varepsilon_{p2} + \nu\varepsilon_{p1}) = \frac{(10,600)}{1-(0.33)^2}[(-700)+0.33(1050)](10^{-6})$$

$\sigma_{p2} = -4.2050$ ksi $\cong$ 4.21 ksi (C) .... **Ans.**

$\sigma_{p3} = 0$ ksi .... **Ans.**

$\tau_{max} = \tau_p = (\sigma_{p1} - \sigma_{p2})/2 = 6.97$ ksi .... **Ans.**

4-32

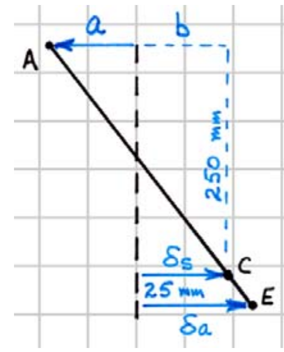
$$\delta = \varepsilon L = \frac{PL}{AE} + \alpha \Delta T L$$

$$\delta_a = 0 + \left(22.5\times10^{-6}\right)(75)(300) = 0.50625 \text{ mm}$$

$$\delta_s = 0 + \left(11.9\times10^{-6}\right)(75)(300) = 0.26775 \text{ mm} = b$$

$$a + b = \frac{250}{25}\left(\delta_a - \delta_s\right) = \frac{250}{25}\left(0.50625 - 0.26775\right)$$

$a = 2.12 \text{ mm} \leftarrow$ .... **Ans.**



4-49

$$E = 29{,}000 \text{ ksi} \qquad G = 11{,}000 \text{ ksi} \qquad E = 2(1+\nu)G$$

$$29{,}000 = 2(1+\nu)(11{,}000) \qquad \nu = 0.31818$$

$$\delta_{xa} = \varepsilon_{xa} L_{xa} = \varepsilon_{xa}(2) = \frac{6 - 0.31818\sigma_y}{29{,}000}(2)$$

$$\delta_{xb} = \varepsilon_{xb} L_{xb} = \varepsilon_{xb}(3) = \frac{5}{29{,}000}(3)$$

But $\delta_{xa} = \delta_{xb}$, therefore

$$12 - 0.63636\sigma_y = 15$$

$\sigma_y = -4.71$ ksi .... **Ans.**

4-51*

Assume series of rails all initially separated by 0.125 in.
When heated, rails expand from center in both directions.

(a) $\delta = 0.125 = \alpha\, \Delta T\, L = \left(6.6\times10^{-6}\right)\left(\Delta T\right)\left(55\times12\right)$

$\Delta T = 28.7\ °\text{F}$

Rails touch when $T = 60 + 28.7 = 88.7\ °\text{F}$ .... **Ans.**

(b) $\delta = \left(6.6\times10^{-6}\right)\left(-50\right)\left(55\times12\right) = -0.21780$ in.

$gap = 0.125 + 0.2178 = 0.3428\ \text{in.} \cong 0.343\ \text{in.}$ .... **Ans.**