Mechanics of Materials

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Chapter 9

Analyzing Columns

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Analyzing Columns

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9-1 Introduction

- Long, straight, prismatic bars subjected to compressive axial loads.
- Begin to deform laterally → ***buckling*** (挫屈, sudden large deformation in lateral direction).
- Not caused by failure of material, but by deterioration of stable state.
- Critical buckling load: the max. load for which the column is in stable equilibrium.
- For long slender column, critical buckling load << stress of proportional limit.

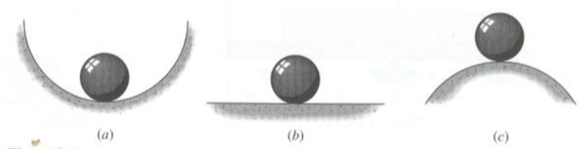

Figure 9-1

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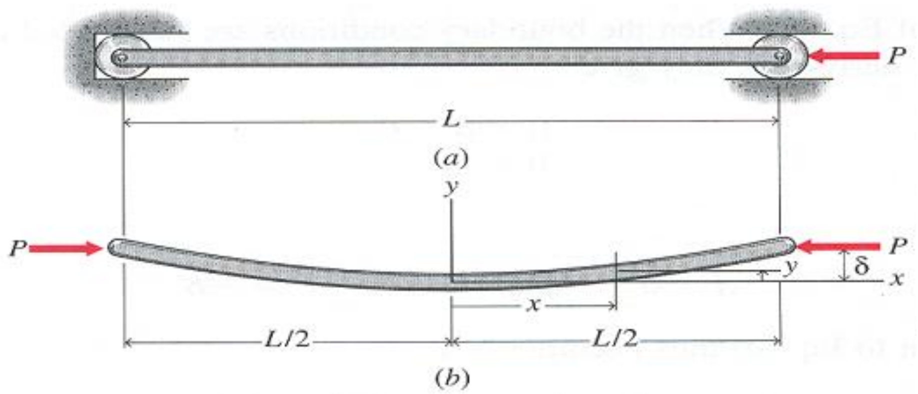
9-2 buckling of long straight columns(I)

- The first solution by Swiss mathematician ***Leonhard Euler*** in 1757



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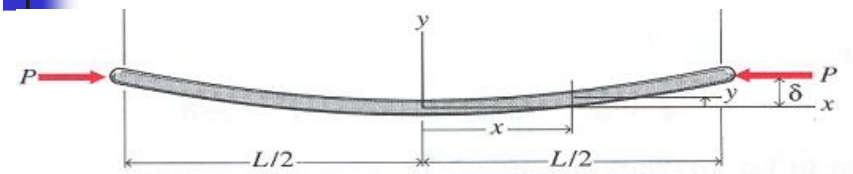
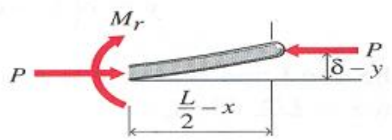
9-2 buckling of long straight columns(II)



(b)



(c)

$$EI\frac{d^2y}{dx^2} = M_r = P(\delta - y)$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{P\delta}{EI} \qquad (*)$$

$$\Rightarrow \quad y = A\sin px + B\cos px + C$$

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9-2 buckling of long straight columns (III)

Substitute y-equation into Eq. (*)

$$\left(-p^2 + \frac{P}{EI}\right)(A\sin px + B\cos px) + \frac{PC}{EI} = \frac{P\delta}{EI}$$

$$\Rightarrow \quad p^2 = \frac{P}{EI} \quad \text{and} \quad C = \delta$$

The constants A and B in $y = A\sin px + B\cos px + \delta$

can be obtained from B.C.'s.

$$\text{at } x = 0, \quad y = 0 \quad \text{and} \quad dx/dy = 0$$

$$\Rightarrow \quad A = 0 \quad \text{and} \quad B = -\delta$$

$$\Rightarrow \quad y = \delta\left(1 - \cos\sqrt{\frac{P}{EI}}x\right)$$

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9-2 buckling of long straight columns (IV)

From $y = \delta\left(1 - \cos\sqrt{\frac{P}{EI}}x\right)$

Since $x = L/2,\ y = \delta$ ⇒ $\cos\sqrt{\frac{P}{EI}}\frac{L}{2} = 0$

$$\sqrt{\frac{P}{EI}}\frac{L}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

The first value has physical significance: **critical bucking load** **(Euler buckling load)**

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad \Rightarrow \quad \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2} = \sigma_{cr}$$

$I = Ar^2$

Slenderness ratio

(r: Radius of gyration about axis of bending)

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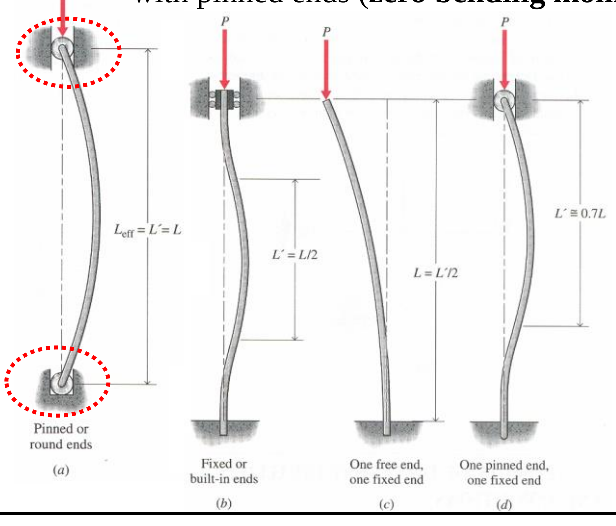
9-3 Effects of different idealized end conditions(I)

- Euler buckling formula was determined for a column with pinned ends (**zero bending moments** at both ends).



L': effective length of the column (distance between two successive inflection points)

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Remarks (I)

- Critical force P_{cr} is valid in the linearly elastic range
- Yield strength is often used instead of proportional limit
- The Euler bucking load (for steel columns) agrees well with experimental data for **L/r > 140**.
- Yielding occurs before buckling for short compression members (**L/r < 40**).
- Between both extremes, neither solution is applicable.

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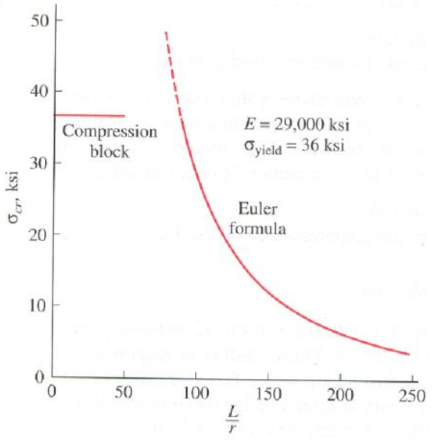
Remarks (II)



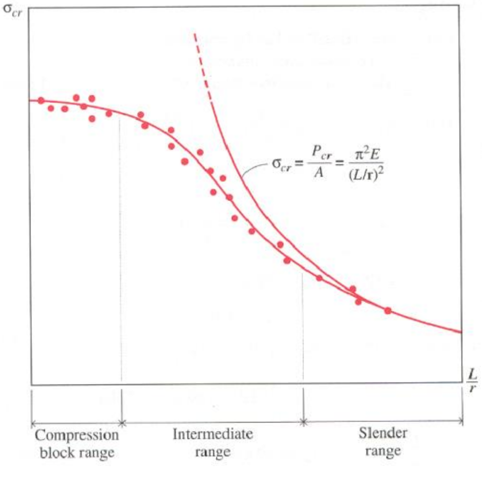


Fig. 9-9 &10

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No Exercises

Chance (God) ***favors*** **the** ***prepared mind.***

Best Luck!