Chapter 3
Fast Scan Techniques

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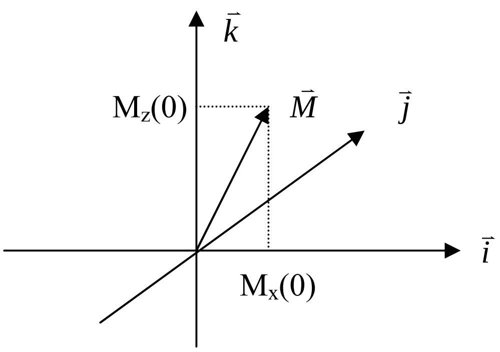
Bloch Eq: $\frac{\partial \vec{M}}{\partial t} = \gamma \vec{M} \times \vec{B}_{eff} - \frac{1}{T_2}\left(M_x \vec{i} + M_y \vec{j}\right) + \frac{1}{T_1}\left(M_0 - M_z\right)\vec{k}$

in rotating frame, and B_1 field has turned off: $\vec{B}_{eff} = 0$

$$\begin{cases} \frac{\partial M_x}{\partial t} = -\frac{1}{T_2} M_x \\ \frac{\partial M_y}{\partial t} = -\frac{1}{T_2} M_y \\ \frac{\partial M_z}{\partial t} = \frac{1}{T_1}\left(M_0 - M_z\right) \end{cases}$$



I.C. : $M_y(0) = 0$, $M_z(\infty) = M_0$

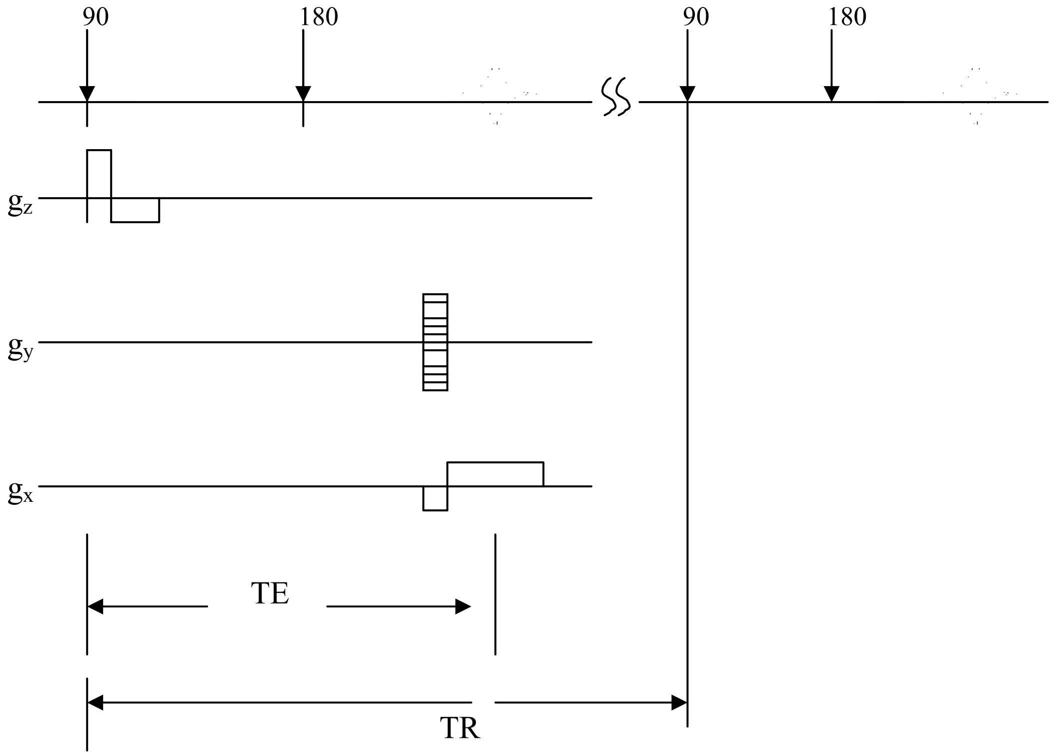
$$\begin{cases} M_x(t) = M_x(0) e^{-t/T_2} \\ M_y(t) = 0 \\ M_z(t) = M_z(0) e^{-t/T_1} + M_0\left(1 - e^{-t/T_1}\right) \end{cases}$$

$\Longrightarrow$ ways of measuring T_1, T_2 by means of various pulse sequences

if $M_z(0) = 0 \Rightarrow M_z(t) = M_0\left(1 - e^{-t/T_1}\right)$

$$M_z(0) = -M_0 \quad \Rightarrow \quad M_z(t) = M_0\left(1 - 2e^{-t/T_1}\right)$$

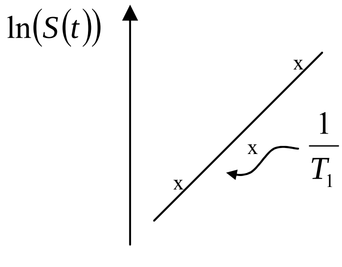
In spin-echo:

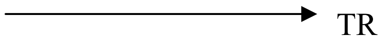


$$S(TR, TE) = M_0\left(1 - e^{-TR/T_1}\right)e^{-TE/T_2}$$

T_1WI: $S(TR) = M_0\left(1 - e^{-TR/T_1}\right)$ $\qquad TE << T_2$

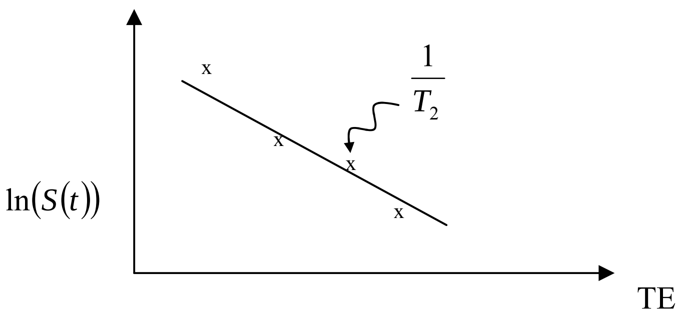
T_1 measurement: $\ln\left(\frac{S(TR) - M_0}{M_0}\right) = \frac{1}{T_1}(TR) = R_1(TR)$





T_2WI: $S(TE) = \left(M_0 e^{-TE/T_2} \right)$ $\qquad TR >> T_1$

T_2 measurement: $\ln\left(\dfrac{S(TE)}{M_0}\right) = -\dfrac{1}{T_2}(TE) = -R_2(TE)$



Now let's consider scan time for conventional SE:

scan time = TR × ϕ-encoding # × NEX

for TR = 500 ms, # phase-encoding = 128, 2 NEX

⇒ scan time ≅ 2 min too slow!

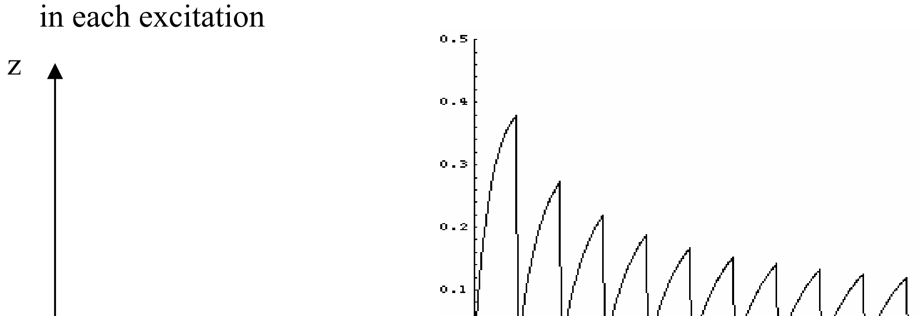
To speed up acquisition, 3 approaches:

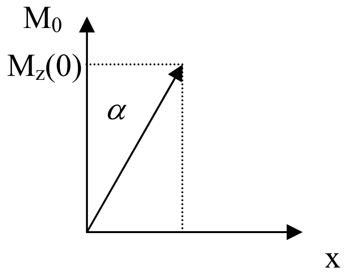
① ↓TR ⇒ use small flip angles, stronger gradient and slew rate GRE, FSE, FGRE, EPI

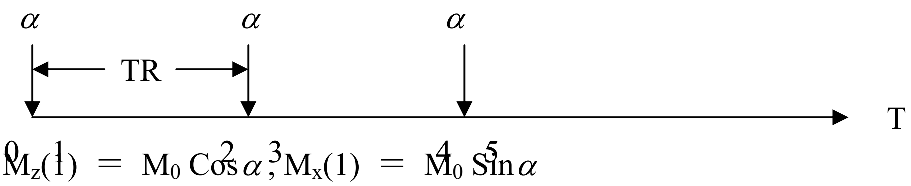
② ↓ϕ-encoding steps ⇒ interpolation, zero-filling, half-Fourier…

③ parallel acquisition ⇒ SENSE, SMASH

① ↓TR: use small flip angle α instead of 90°
steady-state: equilibrium achieved between T_1 relaxation and M_z decrement in each excitation







$M_z(1) = M_0 \mathrm{Cos}\,\alpha$; $M_x(1) = M_0 \mathrm{Sin}\,\alpha$

$M_z(2) = M_0\ (1-e^{-TR/T_1}) + M_0 \mathrm{Cos}\,\alpha\ e^{-TR/T_1}$

$M_z(3) = M_z(2)\ \mathrm{Cos}\,\alpha$

When steady state is achieved:

$M_z(2i) = M_z(2\,(i - 1)) \qquad i > 5$

$M_z(2i) = M_0\,(1 - e^{-TR/T_1}) + M_z(2\,(i - 1))\ \mathrm{Cos}\,\alpha\ e^{-TR/T_1}$

$\qquad = M_0\,(1 - e^{-TR/T_1}) + M_z(2i)\ \mathrm{Cos}\,\alpha\ e^{-TR/T_1}$

$$\Rightarrow M_z(2i) = \frac{M_0\left(1-e^{-TR/T_1}\right)}{1-Cos\alpha\ e^{-TR/T_1}}$$

$M_x = M_z(2i)\ \mathrm{Sin}\,\alpha$

$$\therefore \quad \text{echo} = k\,e^{-TE/T_2^*}\ \frac{M_0\left(1-e^{-TR/T_1}\right)}{1-\cos\alpha\,e^{-TR/T_1}}\ \mathrm{Sin}\,\alpha$$

max M_x when $\frac{dM_x(\alpha)}{d\alpha}=0$, so solve for α, we get

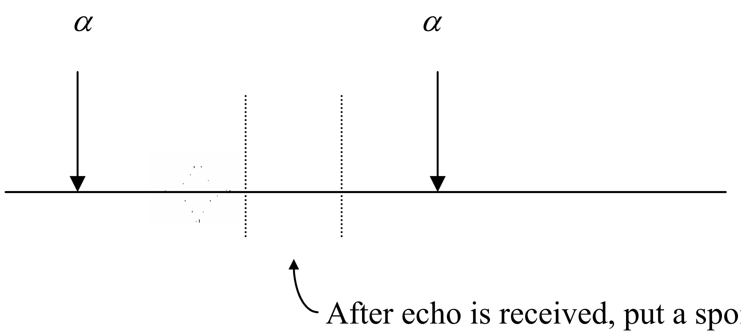
$$\boxed{\alpha = \cos^{-1}\left(e^{-TR/T_1}\right)}$$

"Ernst angle"

$\therefore$ long $T_1 \rightarrow \alpha \sim 0°$ or long $TR \rightarrow \alpha \sim 90°$

short $T_1 \rightarrow \alpha \sim 90°$ or short $TR \rightarrow \alpha \sim 0°$

Next question: spoil or refocus, when TR << T_2 ?



After echo is received, put a spoiler or a rewinder to "clean up" or realign the residual magnetizatin in x-y plane.

The signal in images with spoiler gradients depends on M_z before echo α-pulse and the length of TE after excitation:

$$S = k \frac{M_0\left(1-e^{-TR/T_1}\right)}{1-\cos\alpha\, \mathrm{e}^{-TR/T_1}} \sin\alpha \quad e^{-TE/T_2^*}$$

when $\alpha \rightarrow 0$, $s \sim ke^{-TE/T_2^*} M_0 \alpha$ $\begin{cases} TE \uparrow \Rightarrow T_2^* WI \\ TE \downarrow \Rightarrow PDI \end{cases}$

when $\alpha \rightarrow 90$, $s \sim ke^{-TE/T_2^*} M_0\left(1-e^{-TR/T_1}\right)$ $\begin{cases} TE \uparrow \Rightarrow T_1 \ \& T_2^* ? \\ TE \downarrow \Rightarrow T_1 WI \end{cases}$

Note: α small to suppress T_1 effect, TE large to enhance T_2^* effect.

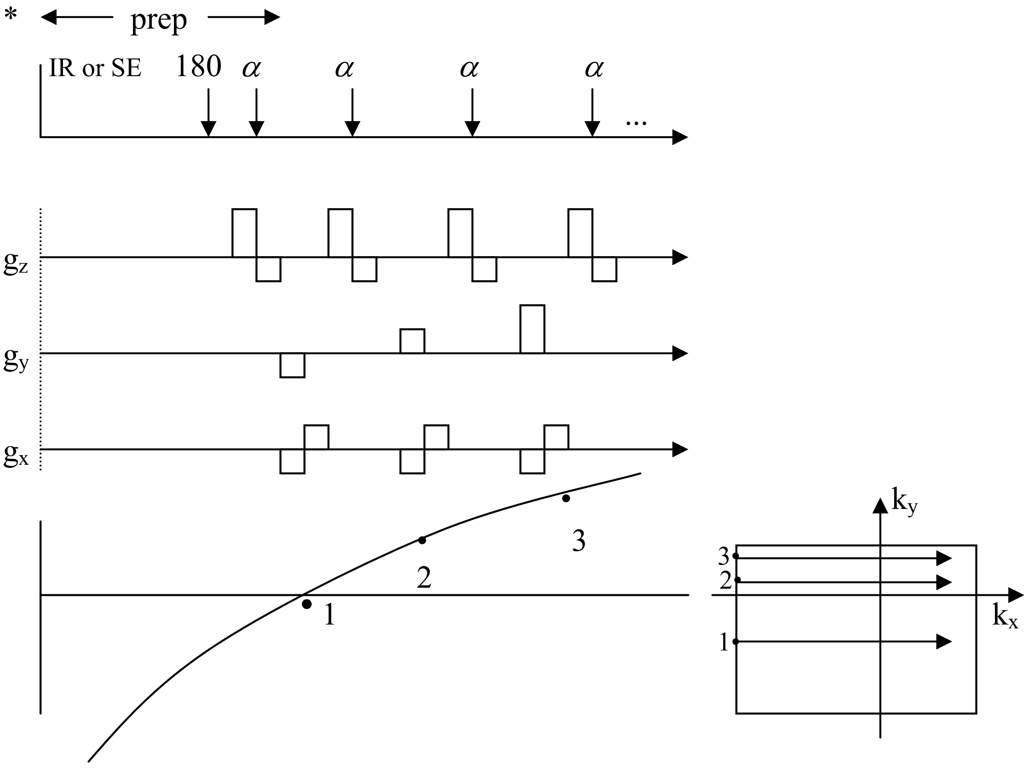
So the image contrast depends on α, TE, not TR!

$TR \approx 20ms.$

Scan time $\approx 20ms \times 128 \times 2 \cong 5\sec.$ vs. 2 min in SE "Turbo FLASH" (MRM 1990;13:77-89)

* $TR \approx 3ms.$, use small flip angles

* Scan time $\approx 3ms. \times 128 \times 2 \approx 770ms.$ (sub-second)
* use preparatory pulses. eg. IR or SE to give image contrast
*



* center k determines image contrast (most of the energy @ center k) peripheral k determines image resolution

Steady-state free precession (SSFP) or steady-state coherent (SSC) imaging

"free" precession: precession angle with no gradients on;

ΔB_0 is the source of resonance offset

$$\beta(t) = \gamma \Delta B(t) + \gamma \vec{r} \cdot \int_0^t \vec{g}(t) d\tau$$
$$= \gamma \Delta B(t)$$
$$= \Delta w(t)$$

For $t' = t - nTR$

$$\begin{cases} M_x(n,t') = \left[M_x^+(n) Cos\beta(t') + M_y^+(n) Sin\beta(t')\right] e^{-t'/T_2} \\ M_y(n,t') = \left[-M_x^+(n) Sin\beta(t') + M_y^-(n) Cos\beta(t')\right] e^{-t'/T_2} \\ M_z(n,t') = M_z^+(n) e^{-t'/T_1} + M_0\left(1 - e^{-t'/T_1}\right) \end{cases}$$

Also, $R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & Cos\theta & Sin\theta \\ 0 & -Sin\theta & Cos\theta \end{pmatrix}$

$$D(t') = \begin{bmatrix} e^{-t'/T_2} Cos\beta(t') & e^{-t'/T_2} Sin\beta(t') & 0 \\ -e^{-t'/T_2} Sin\beta(t') & e^{-t'/T_2} Sin\beta(t') & 0 \\ 0 & 0 & e^{-t'/T_1} \end{bmatrix} \Rightarrow \vec{M}(t') = \underline{D}(t') \vec{M}^+$$

$\therefore \quad \vec{M}^+(n) = R_x(\theta) \vec{M}^-(n)$ ………………………. (1)

$\vec{M}^-(n+1) = D(T_R) \vec{M}^+(n) + M_0(1 - E_1)\hat{k}$ …………… (2)

in steady-state condition: $\vec{M}^-(n+1) = \vec{M}^-(n)$

$$\vec{M}^{-}(n) = D(T_R)\vec{M}^{+}(n-1) + M_0(1-E_1)\hat{K} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (3)$$

Also $\vec{M}^{+}(n) = \vec{M}^{+}(n-1)$

$$\therefore \begin{cases} \vec{M}^{-}(\infty) = (I - D(T_R)R_x(\theta))^{-1} M_0(1-E_1)\hat{k} & (By(1)\ \&\ (2)) \\ \vec{M}^{+}(\infty) = R_x(\theta)(I - R_x(\theta)D(T_R))^{-1} M_0(1-E_1)\hat{k} & (By(1)\ \&\ (3)) \end{cases}$$

$$M_x^{-}(\infty) = M_0(1-E_1)\frac{E_2 Sin\theta Sin\beta}{d}$$

$$M_y^{-}(\infty) = M_0(1-E_1)\frac{E_2 Sin\theta(Cos\beta - E_2)}{d}$$

$$M_z^{-}(\infty) = M_0(1-E_1)\frac{[(1-E_2Cos\beta) - E_2Cos\theta(Cos\beta - E_2)]}{d}$$

$$M_x^{+}(\infty) = M_x^{-}(\infty)$$

$$M_y^{+}(\infty) = M_0(1-E_1)\frac{Sin\theta(1-E_2Cos\beta)}{d}$$

$$M_z^{+}(\infty) = M_0(1-E_1)\frac{[E_2(E_2 - Cos\beta) + (1-E_2Cos\beta)Cos\theta]}{d}$$

$$d = (1-E_1Cos\theta)(1-E_2Cos\beta) - E_2(E_1 - Cos\theta)(E_2 - Cos\beta)$$

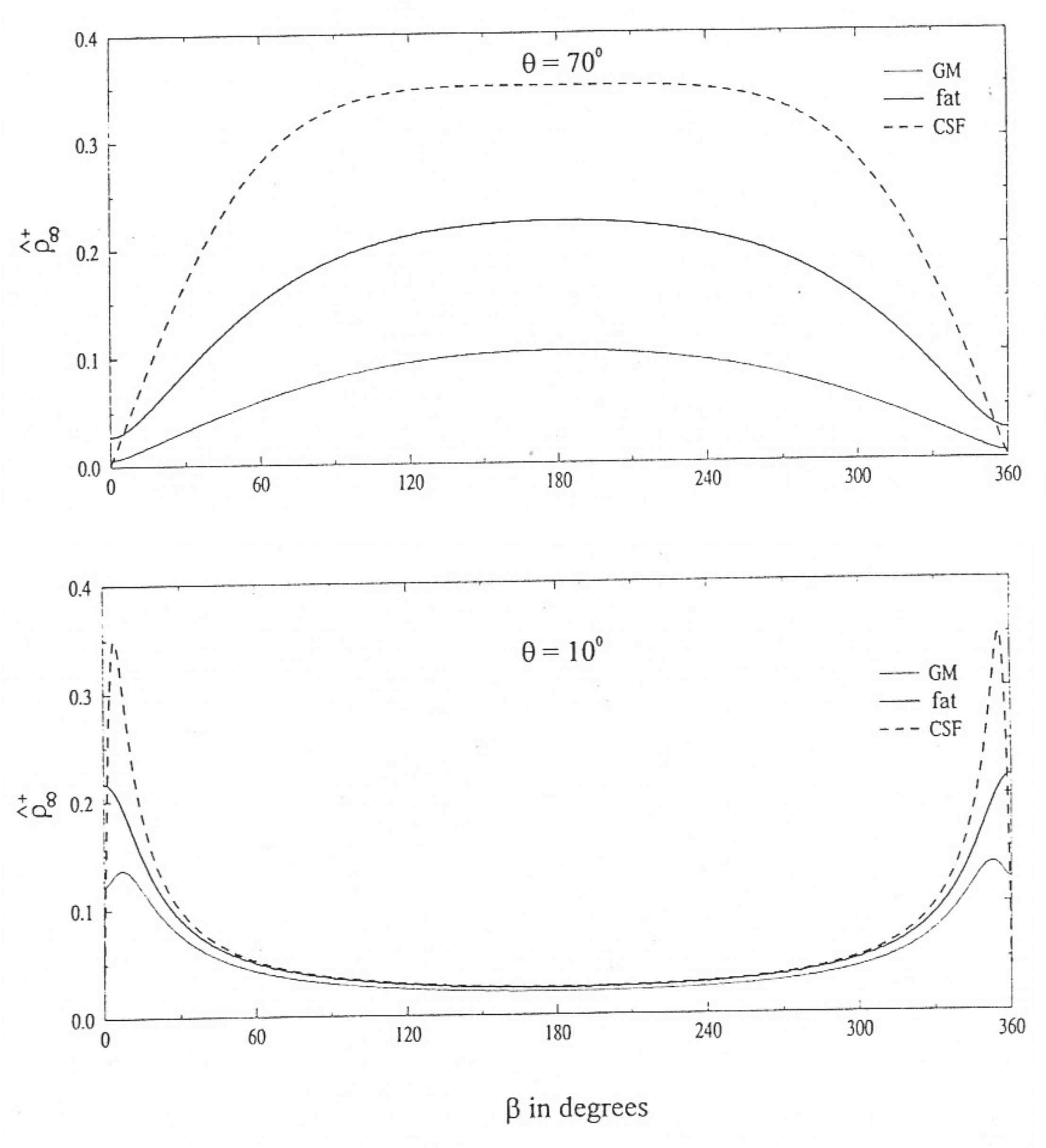
θ: flip angle

β: precession angle in SSFP

$E_1 = e^{-TR/T_1}$

$E_2 = e^{-TR/T_2}$

β



Spatial variation in signal is moderate if $\beta \cong 180^\circ$,

but is drastic if $\beta \cong 0^\circ$ or 360°.

For $TR << T_2, T_1$, $\beta = \pi$

$$M_y^+(\infty) = \frac{M_0(1-E_1)Sin\theta}{(1-E_1Cos\theta)-E_2(E_1-Cos\theta)}$$

$$\cong \frac{M_0 Sin\theta}{\left(\frac{T_1}{T_2}+1\right)-Cos\theta\left(\frac{T_1}{T_2}-1\right)}$$

$$Cos\theta_{opt} = \frac{E_1 + E_2\left(Cos\beta - E_2\right)/\left(1 - E_2 Cos\beta\right)}{1 + E_1 E_2\left(Cos\beta - E_2\right)/\left(1 - E_2 Cos\beta\right)}$$

for $\beta \cong \pi$

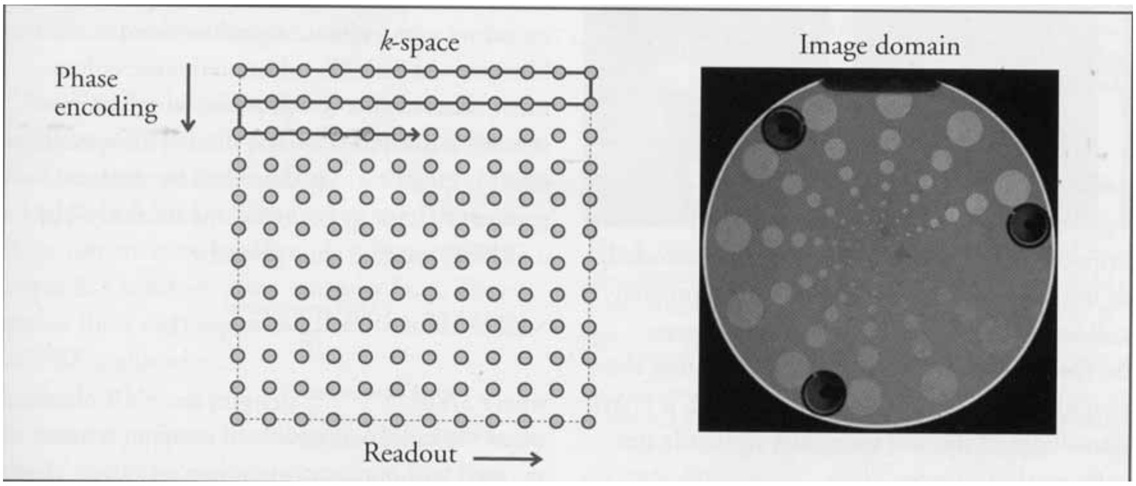
$$Cos\theta_{opt} \cong \frac{T_1/T_2 - 1}{T_1/T_2 + 1}$$

$$M_y^+(\infty)\Big|_{\theta=\theta_{opt}} \cong \frac{1}{2} M_0 \sqrt{\frac{T_2}{T_1}}$$

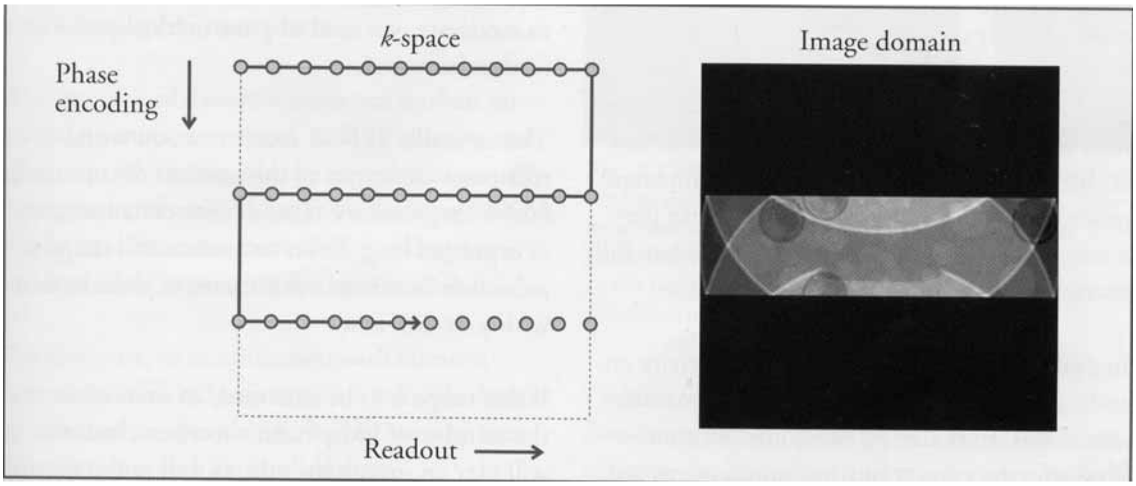
SENSE (sensitivity encoding)

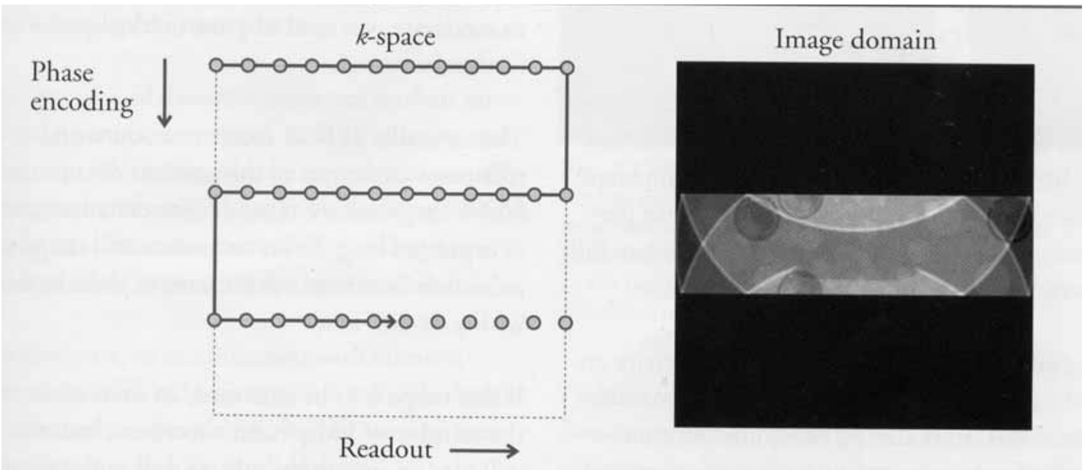
key idea: multiple coils parallel acquisition using fewer phase-encoding steps with larger Δk_y The resulting images from n coils show small FOV with aliasing. Making use of different sensitivities from different coils, the location of image pixel can be determined (unwrapped).

MRM 42:952(1999)
MRM 44:495(2000)



Now use 2 surface coils, with same acquisition scheme:





let $f(y) \Leftrightarrow F(k_y)$ image @ y

$c_n(y) \Leftrightarrow C_n(k_y)$ coil sensitivity

The detected signal by the nth coil:

$$S_n(k_y) = \int dy e^{-2\pi i k_y y} c_n(y) f(y) = C_n(k_y) \otimes F(k_y)$$

$$\equiv \int dk_y^{'} C_n(k_y - k_y^{'}) F(k_y^{'})$$

let N_A = # of views acquired, $A \equiv \{1,2,\cdots N_A\}$

N_R = # of views of reconstructed images $R \equiv \{1,2,\cdots N_R\}$

N_C = # of coils

$(N_R \leq N_A N_C)$

The total acquired data:

$$S_{n,k} = \sum_{k' \in R} C_{n,k-k'} F_{k'} \qquad k \in A, \qquad n \in \{1,2,\cdots\cdots N_c\}$$

a set of $N_A N_C$ linear equations for N_R unknowns

$$\tilde{S} = \tilde{C}\tilde{F}$$

$$\underbrace{N_A N_C}\ \underbrace{N_A N_C \times N_R}\ \underbrace{N_R}$$

if $N_R = N_A N_C$, $\tilde{F} = \tilde{C}^{-1}\tilde{S}$

if $N_R < N_A N_C$, least squares of difference:

$$\min\left\{\sum_{n,k}\left|S_{n,k} - \sum_{k'} C_{n,k-k'}F_{k'}\right|^2 / \sigma_n^2\right\}$$

σ_n : the noise in the n-th coil

$$\Rightarrow \tilde{F} = \left(\tilde{C}^H \tilde{\Psi} \tilde{C}\right)^{-1} \tilde{C}^H \tilde{\Psi}^{-1} \tilde{S}$$

$\left\{\Psi^{-1}\right\}_{nk,n'k'} = \delta_{nn'}\delta_{kk'} / \delta_n^2$ "noise weighting matrix" or "receiver noise matrix"

if similar decoupled coils: $\Psi^{-1} = \tilde{1}$

$\tilde{F} = \left(\tilde{C}^H \tilde{C}\right)^{-1} \tilde{C}^H \tilde{S} = \tilde{C}^{-1}\tilde{S}$, then we get $f(y)$ by $\mathbf{F}\{\mathrm{F}\}$

For Example: 2 coils, $dk_y = 2/FOV_y$

$$\begin{cases} S_{1,2m} = \sum_{k' \in R} C_{1,2m-k'}F_{k'} & m \in \left\{1,2,\cdots\cdots N_R/2\right\} \\ S_{2,2m} = \sum_{k' \in R} C_{2,2m-k'}F_{k'} & m \in \left\{1,2,\cdots\cdots N_R/2\right\} \end{cases}$$

we can **F** $\{S_{1,2m}\}$, **F** $\{S_{2,2m}\}$ first:

$$\begin{cases} s_{1,y} = c_{1,y} f_y + c_{1,y+FOV/2} f_{y+FOV/2} \\ s_{2,y} = c_{2,y} f_y + c_{2,y+FOV/2} f_{y+FOV/2} \end{cases}$$

$$\Rightarrow \begin{cases} f_y = \left(c_{2,y+FOV/2} s_{1,y} - c_{1,y+FOV/2} s_{2,y}\right) / \left(c_{2,y+FOV/2} c_{1,y} - c_{1,y+FOV/2} c_{2,y}\right) \\ f_{y+FOV/2} = \left(c_{1,y} s_{2,y} - c_{2,y} s_{1,y}\right) / \left(c_{2,y+FOV/2} c_{1,y} - c_{1,y+FOV/2} c_{2,y}\right) \end{cases}$$

How to determine coil sensitivity?

$$\frac{I_{Nc}}{I_{ref}}$$

I_{Nc}: full FOV, low resolution image for coil element N_c

I_{ref}: corresponding image with body coil

Basic SMASH Theory

The MR Signal obtained using a standard Fourier imaging sequence may be written as:

$$S(k_x, k_y) = \iint dx dy C(x, y) \rho(x, y) \exp\{-ik_x x - ik_y y\}. \quad [1]$$

Here $\rho(x, y)$ is the spatial spin distribution, and $C(x, y)$ represents the sensitivity of the RF coil used for detection. The SMASH technique exploits the inherent spatial variations in the coil sensitivity profile, $C(x, y)$, to generate multiple phase-encoding lines at once.

If a linear multi-coil array is used for imaging, it can be shown that it is possible to use linear combinations of component coil sensitivities to generate composite sensitivity profiles, which take the form of complex exponentials such that:

$$C_m^{comp}(x, y) = \sum_{l=1}^{L} n_l^{(m)} C_1(x, y) = \exp\{-im\Delta k_y y\}. \quad [2]$$

Here $\Delta k_y = 2\pi/FOV$, l = 1, 2, …..L, where L is the total number of coils in the array, $C_l(x,y)$ is the sensitivity of coil *l*, and $n_l^{(m)}$ is a set of complex weights required to generate a given harmonic, $\exp(-im\Delta k_y y)$, of integer order m. The complex exponentials of Eq. [2] are of the same form as the spatial modulations produced by phase-encoding gradients and therefore produce the same effect.

For profiles of the form of Eq. [2], the MR signal then becomes:

$$S_m^{comp}(k_x,k_y)=\sum_{l=1}^{L} n_l^{(m)} S_l(k_x,k_y)$$

$$=\iint dxdy \sum_{l=1}^{L} n_l^{(m)} C_l(x,y)\rho(x,y)\cdot\exp\{-ik_x x,-ik_y y\}$$

$$=\iint dxdy C_m^{comp}(x,y)\rho(x,y)\cdot\exp\{-ik_x x,-ik_y y\}$$

$$=\iint dxdy \rho(x,y)\cdot\exp\{-ik_x x,-i(k_y+m\Delta k_y)y\}$$

$$=S(k_x,k_y+m\Delta k_y)$$

In other words, linear combinations of component coil signals with appropriate weights produce exactly the same k-space shift that would have been obtained by application of a standard phase-encoding gradient of amplitude $\gamma G_y t=-m\Delta k_y$. Therefore, if a suitable coil array is available, a reduced number of phase-encoding steps may be acquired, and the remainder of the k-space matrix may be filled in after the fact using various linear combinations of component coil signals to produce the appropriate k-space shifts. In single-shot imaging, this corresponds to a decrease in the total acquisition time required for a given image. The impact of this decrease in acquisition time on spatial resolution is the focus of the next section.